$$\boxed{\log_a x = \log_a y \longleftrightarrow x = y}$$

This means two things:

1) If both sides of an equation have a single logarithmic term with the same base, then we can drop the logarithm stuff. This is going from left to right in the box above.
2) We can write both sides of a regular equation as logarithmic terms with the same base. This is going from right to left in the box above. We'll check this out later.

Here are a couple examples of making the logarithms disappear.

Example 1 – Solve for x

$$\log_2 x + \log_2 3 = \log_2 27$$

Step 1- Get single logarithms with the same base on both sides

$$\log_2 x + \log_2 3 = \log_2 27$$
$$\log_2 3x = \log_2 27$$

Step 2- Drop the logarithm stuff

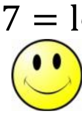
$$\log_2 3x = \log_2 27$$
$$3x = 27$$

Step 3- Solve the resulting equation

$$3x = 27$$
$$x = 9$$

Step 4- Check the solution(s). Look out for logarithms of negative numbers.

$$\log_2(9) + \log_2 3 = \log_2 27$$
$$\log_2 27 = \log_2 27$$



$$\boxed{\boldsymbol{x = 9}}$$

<u>Example 2</u> – Solve for x

$$\log_a(x^3+19) = 3\log_a 3$$

Step 1- Get single logarithms with the same base on both sides

$$\log_a(x^3+19) = 3\log_a 3$$
$$\log_a(x^3+19) = \log_a 3^3$$
$$\log_a(x^3+19) = \log_a 27$$

Step 2- Drop the logarithm stuff

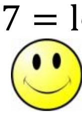
$$\log_a(x^3+19) = \log_a 27$$
$$x^3+19 = 27$$

Step 3- Solve the resulting equation

$$x^3+19 = 27$$
$$x^3 = 8$$
$$x = 2$$

Step 4- Check the solution(s). Look out for logarithms of negative numbers.

$$\log_a((2)^3+19) = 3\log_a 3$$
$$\log_a(8+19) = \log_a 3^3$$
$$\log_a 27 = \log_a 27$$



$$\boxed{\boldsymbol{x = 2}}$$

Sometimes we need to use logarithms on both sides of an equation. This is necessary when

1) the variable is in the exponent position, and
2) we can't easily get the same bases on both sides of the equation (up to this point in the class, we've always been able to get the same bases- but not today!).

When we use the logarithms, we will use the base of 10 (unless the number e is involved, but we'll save that for another day!) because that is what is easily accessible on your calculator.

<u>Example 3</u> – Solve for x. Round your answer to three decimal places.

$$4^x = 73$$

Step 1- Introduce logarithms to both sides of the equation

$$4^x = 73$$
$$\log 4^x = \log 73$$

Step 2- Move the exponent to the front of the logarithm. This will get the variable out of the exponent position and down where we can do something with it!

$$\log 4^x = \log 73$$
$$x \log 4 = \log 73$$

Step 3- Solve for the variable. Have a calculator ready!

$$x \log 4 = \log 73$$

$$x = \frac{\log 73}{\log 4} = \frac{1.86332286}{0.602059991} \approx 3.095$$

$$\boxed{\boldsymbol{x \approx 3.095}}$$

TIP: If the variable *<u>is not</u>* in the exponent position, you *<u>don't need to use logarithms</u>* to solve. If the variable <u>is</u> in the exponent position, you will have to <u>use logarithms</u> to solve if you can't get the same bases on both sides.